INDIAN STATISTICAL INSTITUTE, BANGALORE CENTRE
B.MATH - Third Year, Second Semester, 2010-11
Statistics - IV, Final Examination

1. Suppose we have a random sample $X_1, \ldots, X_n$ from a continuous distribution with c.d.f. F and density f, both of which are completely unknown.
(a) Define the histogram estimate of f.
(b) Show that the histogram is a consistent estimator of f if the interval width is chosen to be $n^{-2/3}$. [10]

2. Suppose $X_1, \ldots, X_n$ are i.i.d. Bernoulli(θ) where $0 < \theta < 1$ is unknown but n is fixed. Consider estimating θ under the loss
$L(\theta, a) = \theta^{-1/2}(1-\theta)^{-1/2}(\theta - a)^2$.
Find the Bayes estimator of θ with respect to the prior
$\pi(\theta) \propto \theta^{3/2}(1-\theta)^{3/2}$, $0 < \theta < 1$. [10]

3. Suppose $X_1, \ldots, X_n$ is a random sample from Poisson(θ), where $\theta > 0$. Consider the loss $L(\theta, a) = (\theta - a)^2/\theta$, where $a \geq 0$. Derive the minimax estimator of θ. [10]

4. Let $X_1, \ldots, X_n$ be a random sample from $N(\theta, \sigma^2)$, where $\theta > 0$ is unknown but σ^2 is known. Consider the decision problem where the loss function is $L(\theta, a) = (\theta - a)^2$. Show that the decision rule $\delta(X_1, \ldots, X_n) = \bar{X}$ is inadmissible as an estimator of θ. [10]

5. Solve the 2-person, zero-sum game with the following loss matrix:

	a_1	a_2	a_3	a_4
θ_1	2	1	0	0
θ_2	0	1	2	1
θ_3	0	2	0	2

[10]